Time Value of Money - Sample Problems

1. If you wish to accumulate $140,000 in 13 years, how much must you deposit today in an account that pays an annual interest rate of 14%?

2. What will $247,000 grow to be in 9 years if it is invested today in an account with an annual interest rate of 11%?

3. How many years will it take for $136,000 to grow to be $468,000 if it is invested in an account with an annual interest rate of 8%?

4. At what annual interest rate must $137,000 be invested so that it will grow to be $475,000 in 14 years?

5. If you wish to accumulate $197,000 in 5 years, how much must you deposit today in an account that pays a quoted annual interest rate of 13% with semi-annual compounding of interest?

6. What will $153,000 grow to be in 13 years if it is invested today in an account with a quoted annual interest rate of 10% with monthly compounding of interest?

7. How many years will it take for $197,000 to grow to be $554,000 if it is invested in an account with a quoted annual interest rate of 8% with monthly compounding of interest?

8. At what quoted annual interest rate must $134,000 be invested so that it will grow to be $459,000 in 15 years if interest is compounded weekly?

9. You are offered an investment with a quoted annual interest rate of 13% with quarterly compounding of interest. What is your effective annual interest rate?

10. You are offered an annuity that will pay $24,000 per year for 11 years (the first payment will occur one year from today). If you feel that the appropriate discount rate is 13%, what is the annuity worth to you today?

11. If you deposit $16,000 per year for 12 years (each deposit is made at the end of each year) in an account that pays an annual interest rate of 14%, what will your account be worth at the end of 12 years?

12. You plan to borrow $389,000 now and repay it in 25 equal annual installments (payments will be made at the end of each year). If the annual interest rate is 14%, how much will your annual payments be?

13. You are told that if you invest $11,000 per year for 23 years (all payments made at the end of each year) you will have accumulated $366,000 at the end of the period. What annual rate of return is the investment offering?

14. You are offered an annuity that will pay $17,000 per year for 7 years (the first payment will be made today). If you feel that the appropriate discount rate is 11%, what is the annuity worth to you today?

15. If you deposit $15,000 per year for 9 years (each deposit is made at the beginning of each year) in an account that pays an annual interest rate of 8%, what will your account be worth at the end of 9 years?

16. You plan to accumulate $450,000 over a period of 12 years by making equal annual deposits in an account that pays an annual interest rate of 9% (assume all payments will occur at the beginning of each year). What amount must you deposit each year to reach your goal?

17. You are told that if you invest $11,100 per year for 19 years (all payments made at the beginning of each year) you will have accumulated $375,000 at the end of the period. What annual rate of return is the investment offering?

18. You plan to buy a car that has a total "drive-out" cost of $25,700. You will make a down payment of $3,598. The remainder of the car's cost will be financed over a period of 5 years. You will repay the loan by making equal monthly payments. Your quoted annual interest rate is 8% with monthly compounding of interest. (The first payment will be due one month after the purchase date.) What will your monthly payment be?

19. You are considering leasing a car. You notice an ad that says you can lease the car you want for $477.00 per month. The lease term is 60 months with the first payment due at inception of the lease. You must also make an additional down payment of $2,370. The ad also says that the residual value of the vehicle is $20,430. After much research, you have concluded that you could buy the car for a total "drive-out" price of $33,800. What is the quoted annual interest rate you will pay with the lease?

20. You are valuing an investment that will pay you $12,000 the first year, $14,000 the second year, $17,000 the third year, $19,000 the fourth year, $23,000 the fifth year, and $29,000 the sixth year (all payments are at the end of each year). What it the value of the investment to you now is the appropriate annual discount rate is 11.00%?

21. You are valuing an investment that will pay you $27,000 per year for the first ten years, $35,000 per year for the next ten years, and $48,000 per year the following ten years (all payments are at the end of each year). If the appropriate annual discount rate is 9.00%, what is the value of the investment to you today?

22. John and Peggy recently bought a house. They financed the house with a $125,000, 30-year mortgage with a nominal interest rate of 7 percent. Mortgage payments are made at the end of each month. What total dollar amount of their mortgage payments during the first three years will go towards repayment of principal?

23. You are valuing an investment that will pay you $26,000 per year for the first 9 years, $34,000 per year for the next 11 years, and $47,000 per year the following 14 years (all payments are at the end of each year). Another similar risk investment alternative is an account with a quoted annual interest rate of 9.00% with monthly compounding of interest. What is the value in today's dollars of the set of cash flows you have been offered?

24. You have just won the Georgia Lottery with a jackpot of $40,000,000. Your winnings will be paid to you in 26 equal annual installments with the first payment made immediately. If you feel the appropriate annual discount rate is 8%, what is the present value of the stream of payments you will receive?

25. You have just won the Georgia Lottery with a jackpot of $11,000,000. Your winnings will be paid to you in 26 equal annual installments with the first payment made immediately. If you had the money now, you could invest it in an account with a quoted annual interest rate of 9% with monthly compounding of interest. What is the present value of the stream of payments you will receive?

26. You are planning for retirement 34 years from now. You plan to invest $4,200 per year for the first 7 years, $6,900 per year for the next 11 years, and $14,500 per year for the following 16 years (assume all cash flows occur at the end of each year). If you believe you will earn an effective annual rate of return of 9.7%, what will your retirement investment be worth 34 years from now?

27. You plan to retire 33 years from now. You expect that you will live 27 years after retiring. You want to have enough money upon reaching retirement age to withdraw $180,000 from the account at the beginning of each year you expect to live, and yet still have $2,500,000 left in the account at the time of your expected death (60 years from now). You plan to accumulate the retirement fund by making equal annual deposits at the end of each year for the next 33 years. You expect that you will be able to earn 12% per year on your deposits. However, you only expect to earn 6% per year on your investment after you retire since you will choose to place the money in less risky investments. What equal annual deposits must you make each year to reach your retirement goal?

Solutions to Sample Problems
(These solutions assume you have periods per year set equal to one.)

1. n = 13
i = 14
FV = 140000
solve for PV   (answer = $25,489.71)

2. n = 9
i = 11
PV = -247000
solve for FV   (answer = $631,835.12)

3. i = 8
PV = -136000
FV = 468000
solve for n   (answer = 16.06 years)

4. n=14
PV = -137000
FV = 475000
solve for i   (answer = 9.29%)

5. n = 10   (5 years times 2 comp. periods per year)
i = 6.5   (13% annually divided by 2 comp. period per year)
FV = 197000
solve for PV   (answer = $104,947.03)

6. n = 156   (13 years times 12 comp. periods per year)
i = 0.833333   (10% annually divided by 12 comp. periods per year)
PV = -153,000
solve for FV   (answer = $558,386.38)

7. i = 0.666667   (8% annually divided by 12 comp. periods per year)
PV = -197000
FV = 554000
solve for n   (answer on calculator = 155.61)
Since the interest rate was entered as a monthly rate, the answer for n is in months.
The number of years equals the number of months divided by twelve.
Number of years = (155.61)/12 = 12.97 years

8. n = 780   (15 years times 52 comp. periods per year)
PV = -134,000
FV = 459,000
solve for i   (answer on calculator = 0.157972)
Since the number of periods was entered as weeks, the answer for i is the weekly rate.
The annual rate equals the weekly rate times 52.
Annual rate = (0.157972%)(52) = 8.21%

9. n = 4   (number of comp. periods in one year)
i = 3.25   (13% annually divided by 4 comp. periods in one year)
PV = -100
solve for FV   (answer = 113.65)
Subtract the 100 (percent) you initial had to get the EAR.
EAR = 113.65 – 100 = 13.65%

10. n = 11
i = 13
PMT = -24000
Make sure you are in end mode.
solve for PV   (answer = $136,486.59)

11. n = 12
i = 14
PMT = 16000
Make sure you are in end mode.
solve for FV   (answer = $436,331.98)

12. n = 25
i = 14
PV = -389000
Make sure you are in end mode.
solve for PMT   (answer = $56,598.88)

13. n = 23
FV = 366000
PMT = -11000
Make sure you are in end mode.
solve for i   (answer = 3.21%)

14. n = 7
i = 11
PMT = 17000
Make sure you are in begin mode.
solve for PV   (answer = $88,919.14)

15. n = 9
i = 8
PMT = 15000
Make sure you are in begin mode.
solve for FV   (answer = $202,298.44)

16. n = 12
i = 9
FV = 450000
Make sure you are in begin mode.
solve for PMT   (answer = $20,497.98)

17. n = 19
FV = 375000
PMT = -11100
Make sure you are in begin mode.
solve for i   (answer = 5.48%)

18. n = 60   (5 years times 12 payments per year)
i = 0.6667   (8% annually divided by 12 payments per year)
PV = -22102 ($25,700 price minus down payment of $3,598)
Make sure you are in end mode.
solve for PMT   (answer = $448.15)

19. n=60   (total number of payments)
PV = -31430 (price of $33,800 minus $2,370 down payment)
FV = 20,430 (residual value)
PMT = 477
Make sure you are in begin mode.
solve for i   (answer on calculator = 1.122834%)
Since the number of periods was entered as months, the answer for i is the monthly rate.
The annual rate equals the monthly rate times 12.
Annual rate = (1.122834%)(12) = 13.47%

20. CF0 = 0
C01 = 12000
C02 = 14000
C03 = 17000
C04 = 19000
C05 = 23000
C06 = 29000
i = 11
NPV = $76,273.63

TI83:   npv(11,0,{12000,14000,17000,19000,23000,29000})

21. CF0 = 0
C01 =   27000
F01(N$_j$) =   10
C02 =   35000
F02(N$_j$) =   10
C03 =   48000
F03(N$_j$) =   10
i = 9
NPV = $323,123.04

TI83: npv(9,0,{27000,35000,48000},{10,10,10})

22.
First, determine the monthly payment.
n = 360   (30 years times 12 payments per year)
i = 0.5833   (7% annually divided by 12 payment per year)
PV = 125000
Make sure you are in end mode.
solve for PMT   (answer = $831.6281)

Second, solve for the outstanding principal after three years.
n = 324   (360 total payments minus 36 payments made)
i = 0.5833   (7% annually divided by 12 payment per year)
PMT = 831.6281
Make sure you are in end mode
solve for PV   (answer = $120,908.70)

Principal repaid = starting balance minus current balance
Principal repaid = $125,000 - $120,908.70 = $4,091.30

Interest paid = total of payments made – principal repaid
Interest paid = (36)($831.6281) - $4,091.30 = $29,938.61 - $4,091.30 = $25,847.31

23.
Since the payments occur annually, but the interest is compounded monthly, we first must calculate the effective annual interest rate.

n = 12   (number of comp. periods in one year)
i = 0.75   (9% annually divided by 12 comp. periods in one year)
PV = -100
solve for FV   (answer = 109.3807)
Subtract the 100 (percent) you initial had to get the EAR.
EAR = 109.3807 – 100 = 9.3807%

Now calculate the PV of the cash flows using the EAR as the discount rate.
CF0 = 0
C01 =   26000
F01(N$_j$) =   9
C02 =   34000
F02(N$_j$) =   11
C03 =   47000
F03(N$_j$) =   14
i = 9.3807
NPV = $314,517.85

TI83: npv(9.3807,0,{26000,34000,47000},{9,11,14})

24.
n = 26
i = 8
PMT = (40,000,000)/(26)
Make sure you are in begin mode.
solve for PV   (answer = $17,961,194.14)

25. Since the payments occur annually, but the interest is compounded monthly, we first must calculate the effective annual interest rate.

n = 12   (number of comp. periods in one year)
i = 0.75   (9% annually divided by 12 comp. periods in one year)
PV = -100
solve for FV   (answer = 109.3807)
Subtract the 100 (percent) you initial had to get the EAR.
EAR = 109.3807 – 100 = 9.3807%
Now calculate the PV of the cash flows using the EAR as the discount rate.
n = 26
i = 9.3807
PMT = (11,000,000)/(26)
Make sure you are in begin mode.
solve for PV   (answer = $4,453,789.97)

26. Since we do not have a NFV key, we have to solve this problem in two steps. First, calculate the PV of the uneven cash flows. Second, calculate the future value as a lump sum problem.

CF0 = 0
C01 =   4200
F01(N$_j$) =   7
C02 =   6900
F02(N$_j$) =   11
C03 =   14500
F03(N$_j$) =   16
i = 9.7
NPV = $66,239.9844

TI83: npv(9.7,0,{4200,6900,14500},{7,11,16})

n = 34
i = 9.7
PV = -66239.9844
solve for FV   (answer = $1,542,217.26)

27. You must solve this problem in two steps. First, calculate the PV at the time of retirement of the amount needed to give you the annuity and remaining sum wanted. Second, calculate the payment necessary each year over the period from now until retirement to generate the goal.

n = 27
i = 6
FV = 2500000
PMT = 180000
solve for PV   (answer: = $3,038,989.79)
(make sure you are in begin mode)

n = 33
i = 12
FV = 3038989.79
solve for PMT   (answer: = $8,874.79)
(make sure you are in end mode)